## Bay Area Differential Geometry Seminar

Saturday, April 15, 2017, 10 AM-5 PM UC Davis, Mathematical Sciences Building, Room 1147

10:00–11:00: Reception, Morning Coffee

11:00-12:00: Richard Bamler (UC Berkeley) Uniqueness of weak solutions to the Ricci flow

12:00–2:00 Lunch There are many lunch spots in downtown Davis, a 15 minute walk from the Math. Building.

1:45-2:00 Organizational meeting to plan next BADGS

2:00-3:30 Grad student talks:

2:00: Michael Lindsey (UC Berkeley) *Optimal transport via a Monge-Ampere optimization problem*2:30: Gabriel Martins (UCSC) *Magnetic traps in 2 dimensions*3:00 Subhadip Dey (UC Davis) *Spherical metrics with conical singularities on the 2-sphere*

3:30-4:00 Afternoon Tea-Coffee

4:00–5:00 Greg Kuperberg (UC Davis) The Cartan-Hadamard Problem and the Little Prince

5:45 Dinner: Participants and significant others are invited to a dinner to be arranged at a local restaurant on Saturday evening. The cost of the dinner will be reduced or free for participants. (Please email <u>hass@math.ucdavis.edu</u> if you are coming)

Directions to the Math department are at www.math.ucdavis.edu/about/directions

Parking is unrestricted on weekends. The most convenient lot is P46. See taps.ucdavis.edu/sites/taps.ucdavis.edu/files/attachments/parking\_map.pdf

## Abstracts

**Richard Bamler** Uniqueness of weak solutions to the Ricci flow Abstract: In his resolution of the Poincaré and Geometrization Conjectures, Perelman constructed Ricci flows in which singularities are removed by a surgery process. His construction depended on various auxiliary parameters, such as the scale at which surgeries are performed. At the same time, Perelman conjectured that there must be a canonical flow that automatically "flows through its surgeries", at an infinitesimal scale.

Recently, Kleiner and Lott constructed so-called Ricci flow space-times, which exhibit this desired behavior. In this talk, I will first review their construction. I will then present recent work of Bruce Kleiner and myself, in which we show that these Ricci flow space-times are in fact unique and fully determined by their initial data. Therefore, these flows can be viewed as "canonical", hence confirming Perelman's Conjecture. I will also discuss further applications of this uniqueness statement.

**Michael Lindsey** *Optimal transport via a Monge-Ampere optimization problem* Abstract: We rephrase Monge's optimal transportation (OT) problem with quadratic cost---via a Monge--Ampere equation---as an infinite-dimensional optimization problem, which is in fact a convex problem when the target is a logconcave measure with convex support. We define a natural finite-dimensional discretization to the problem and associate a piecewise affine convex function to the solution of this discrete problem. We show that under suitable regularity conditions the convex functions retrieved from the discrete problems converge to the convex solution of the original OT problem furnished by Brenier's theorem. We demonstrate numerical solutions of these discrete problems and then discuss (without proof) how to remove the restriction on the target measure via a fixed point method that only involves solving OT problems with constant target densities.

**Subhadip Dey** Spherical metrics with conical singularities on the 2-sphere Abstract: Given n angles 2\pi\theta\_1, 2\pi\theta\_2, ..., 2\pi\theta\_n, does there exist a spherical metric on the 2-sphere having n singularities of conical type of angles 2\pi\theta\_1, 2\pi\theta\_2, ..., 2\pi\theta\_n? When n=2 or 3, the answer has been known for a long time. Progress has been made in the general case in the past few years. I will discuss some of these results.

## Gabe Martins Magnetic traps in 2 dimensions

Abstract: We study the motion of a charged particle in a bounded region in the plane under the influence of a magnetic field. We show that as long as the field diverges to infinity ``fast enough" at the boundary, the particle cannot reach the boundary in finite time. As a corollary we obtain that the magnetic flow of the field is complete. We then analyze a completely integrable example for which the classical system is complete but its analogous quantum system is not.

**Greg Kuperberg** *The Cartan-Hadamard Problem and the Little Prince* Abstract: Among n-dimensional regions with fixed volume, which one has the least boundary? This question is known as an isoperimetric problem; its nature depends on what is meant by a "region". I will discuss variations of an isoperimetric problem known as the generalized Cartan-Hadamard conjecture: If  $\Omega$  is a region in a complete, simply connected n-manifold with curvature bounded above by  $\kappa \leq 0$ , then does it have the least boundary when the curvature equals  $\kappa$ and  $\Omega$  is round? This conjecture was proven when n = 2 by Weil and Bol; when n = 3 by Kleiner, and when n = 4 and  $\kappa = 0$  by Croke. In joint work with Benoit Kloeckner, we generalize Croke's result to part of the case  $\kappa < 0$ , and we establish a theorem for  $\kappa > 0$ . It was originally inspired by the problem of finding the optimal shape of a planet to maximize gravity at a single point, such as the place where the Little Prince stands on his own small planet.